$\boxed{5.1\ a.}$

eq. 6.25

$$\exp\left(\sum_{n=1}^{\infty}\frac{(ik)^n}{n!}c_n\right)=\sum_{n=1}^{\infty}\frac{(ik)^n}{n!}\langle x^n\rangle$$

$$1+\sum_{n=1}^{\infty}\frac{(ik)^n}{n!}c_n+\frac{1}{2}\left(\sum_{n=1}^{\infty}\frac{(ik)^n}{n!}c_n\right)^2+\frac{1}{6}\left(\sum_{n=1}^{\infty}\frac{(ik)^n}{n!}c_n\right)^3+\dots$$

$$=1+ik\langle x\rangle-\frac{k^2}{2}\langle x\rangle-\frac{ik^3}{6}$$

group terms by order of k

$$1+ikc_1-k^2\left(\frac{1}{2}c_2+\frac{1}{2}c_1^2\right)$$

$$-ik^3\left(\frac{1}{6}c_3+\frac{1}{2}c_1c_2+\frac{1}{6}c_1^3\right)+\dots$$

$$=1+ik\langle x\rangle-\frac{k^2}{2}\langle x^2\rangle-\frac{ik^3}{6}$$

1st

$$\cancel{i}kc_1=\cancel{i}k\langle x\rangle \quad \therefore \quad c_1=\langle x\rangle$$

1st cumulant is just the mean

2nd

$$c_2+c_1^2=\langle x^2\rangle \quad \therefore \quad c_2=\langle x^2\rangle-\langle x\rangle^2$$

$=\langle x\rangle^2$

found above $\langle x^3 \rangle$ variance

$$\cancel{-ik^3}\left(\frac{1}{6}c_3 + \frac{1}{2}c_1c_2 + \frac{1}{6}c_1^3\right) = \cancel{-\frac{ik^3}{6}}\langle x^3\rangle$$

$$c_3 + 3c_1c_2 + c_1^3 = \langle x^3\rangle$$

$$c_3 = \langle x^3\rangle - 3\langle x\rangle\left(\langle x^2\rangle - \langle x\rangle^2\right) - \langle x\rangle^3$$

3rd

c_3 $$= \langle x^3\rangle - 3\langle x\rangle\langle x^2\rangle + 2\langle x\rangle^3$$

God knows what this is...

wikipedia says "skewness"

$\boxed{\text{S.1 B.}}$

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\left(\frac{x-\bar{x}}{2\sigma}\right)^2}$$

probability for a gaussian

C1

intuitively if $c_1 = \langle x\rangle$ we know it will be the mean μ.

C2

$$c_2 = \langle x^2\rangle - \langle x\rangle^2$$

let's value

$$\sigma^2 = \langle x^2\rangle - \langle x\rangle^2$$

solve for $\langle x^2 \rangle$ $\quad \therefore \langle x^2 \rangle = \sigma^2 + \mu^2$

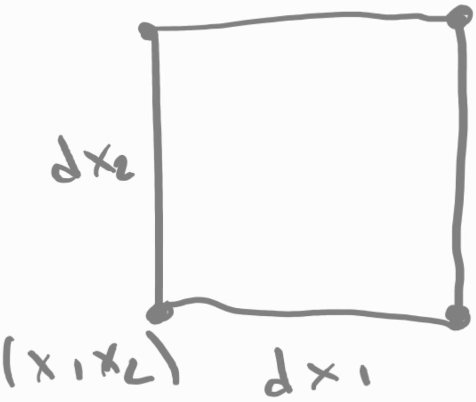
plugging back in $\quad c_2 = \sigma^2 + \mu^2 - \mu^2 = \sigma^2$

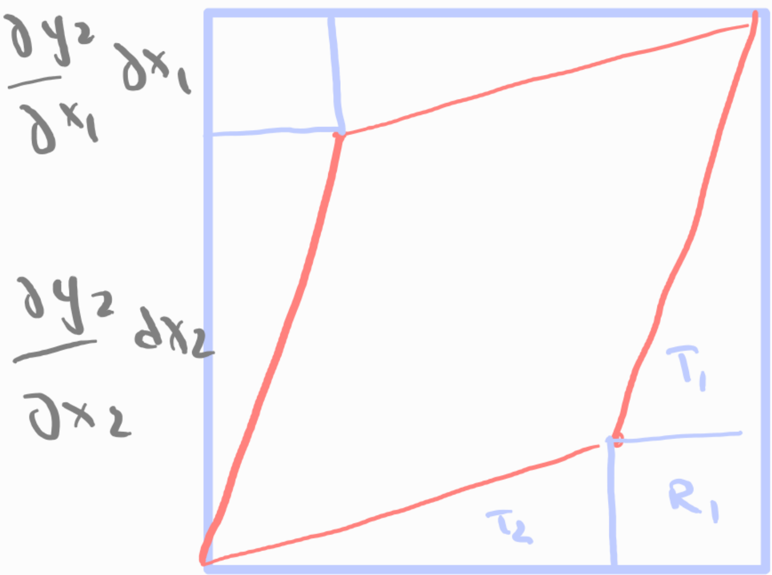
c_3 $\quad c_3 = \langle x^3 \rangle - 3\langle x \rangle \langle x^2 \rangle + 2\langle x \rangle^3$

we need $\langle x^3 \rangle$

$\boxed{5.2\ a}$ $\quad \vec{y}(\vec{x}) = \left(y_1(x_1, x_2),\ y_2(x_1, x_2)\right)$

The parallelogram hint pushes us towards the geometric jacobian below





The blue box has area: $\quad \frac{\partial y_1}{\partial x_1} \partial x_1 \quad \frac{\partial y_1}{\partial x_2} \partial x_2$

$$\left(\frac{\partial y_2}{\partial x_1} \partial x_1 + \frac{\partial y_2}{\partial x_2} dx_2\right)\left(\frac{\partial y_1}{\partial x_1} \partial x_1 + \frac{\partial y_1}{\partial x_2} \partial x_2\right) = R_{outer}$$

The triangles $T_1 + T_2$ & rectangles R_1 $\quad 2(T_1 + T_2$

$+R_1)$

$$2\frac{\partial y_1}{\partial x_2}\partial x_2\frac{\partial y_2}{\partial x_1}\partial x_1+\frac{\partial y_1}{\partial x_1}\partial x_1\frac{\partial y_2}{\partial x_1}\partial x_1+\frac{\partial y_2}{\partial x_2}\partial x_2\frac{\partial y_1}{\partial x_1}\partial x_2$$

without writing it out, the area of the parallelogram is $R_{outer} - R_2(T_1+T_2+R_1)$

Another interpretation would be that the area changes by the determinant of the Jacobian

$\boxed{5.2\ b}$ now if:

$$y_1=\sqrt{-2\ln x_1}\,\sin(x_2)$$
$$y_2=\sqrt{-2\ln x_1}\,\cos(x_2)$$

5.3 9

LFSR = Linear Feedback Shift Register

A register of bits where we shift a value in by XORing from chosen taps.

Table 7.1 says for a 4th order maximal LFSR, tap @ 1+4

so starting with:

XOR

$0 \rightarrow 0 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow$ State

		tap			tap
1	0	0	1	0	0
2	0	0	0	1	0
3	1	0	0	0	1
4	1	1	0	0	0
5	1	1	1	0	0
6	1	1	1	1	0
7	0	1	1	1	1

8	1	0	1	1	1
9	0	1	0	1	1
10	1	0	1	0	1
11	1	1	0	1	0
12	0	1	1	0	1
13	0	0	1	1	0
14	1	0	0	1	1
15	0	1	0	0	1
16	0	0	1	0	0

REPEATS every $2^n - 1$

$2^4 - 1 = 15$

ds

$\boxed{5.3\text{b}}$

GHz years seconds in a year

$$(2^n - 1) \cdot 10^{-9} = 10^{10} \cdot 3.15e^{7}$$

wolfram says: 84.7036 (easy for a computer!)

$\boxed{5.4\ \text{a}}$

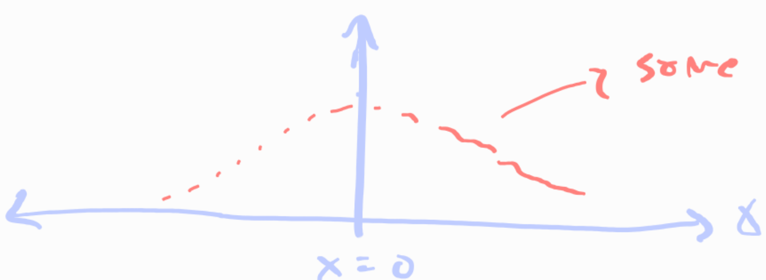
EQ. 7.57

$$\frac{\partial p}{\partial t}(x,t) = D \frac{\partial^2 p}{\partial x^2}(x,t)$$

initial condition is normalized δ function @ origin

@ t = 0

some time later



S.4 d e e

are in python